2.3 Application to Spanning and Linear Independence

Quote. "God exists since mathematics is consistent, and the devil exists since we cannot prove the consistency" Morris Kline (1908-1992)

Vocabulary.

• Dimension: the number of linearly independent vectors required to space.

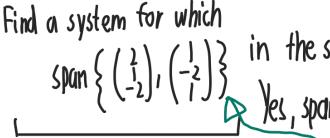
1. Purpose of this section

There will not be anything new in this section. By carefully going through some selected examples, we will see how to use Gaussian elimination and the rank calculation to answer some of the questions we earlier saw about spanning sets, linear independence, and bases.

2. Determining if a given vector is in the span of a set of other vectors.

(onsistent, 0 free variables rank=1, n=3 V_2 (span(B) $V_2 = 3\vec{b}_1 - \vec{b}_2 - \vec{b}_3$

3. Taking a space or subspace defined as a span of some vectors and identifying the space and the system of linear equations that defines the space.



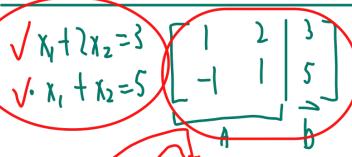
$$A_{\chi}^{2} = 0 \quad \begin{pmatrix} \chi_{1} \\ \lambda_{1} \\ \chi_{3} \end{pmatrix} = t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + s \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix}
1 & 1 & | X_1 \\
1 & -2 & | X_2 \\
-1 & 1 & | X_3
\end{bmatrix}$$

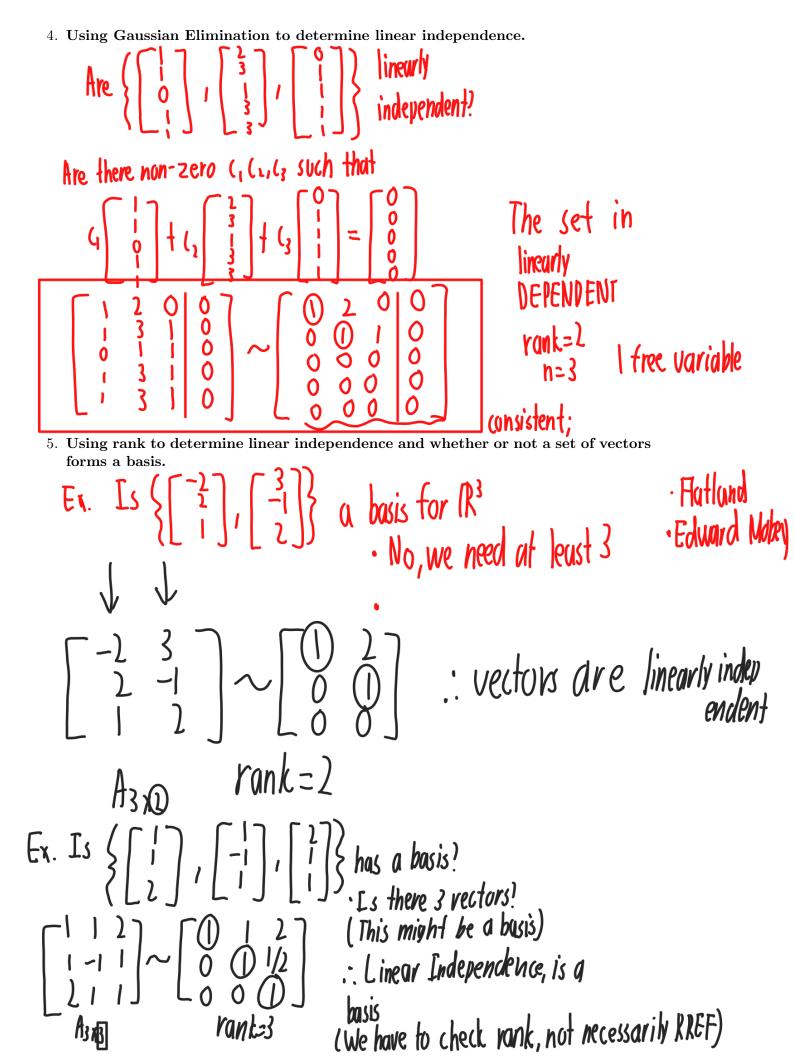
$$\begin{bmatrix}
1 & -2 & | X_2 \\
2 & 1 & | X_3 \\
-1 & 1 & | X_3
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -2 & \chi_2 \\
0 & 1 & 1/5 & (\chi_1 - 1)\chi_2 \\
0 & 0 & 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\
0 & 0 & 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\
\end{bmatrix}$$

$$1/5x_1 + \frac{3}{5}x_2 + x_3 = 0$$



$$y_1 = m_1 x_1 + b_1$$
(don't know why
it's b, always b)
$$y_1 = m_2 x_1 + b_2$$



6. Dimension

The **dimension** of a space is the number of vectors in the basis for that space. The dimension of the trivial subspace (the zero vector) is zero.

If
$$\{[0], [n]\}$$
 is a basis

then dimension of the space Spanned=2

and it is in $[R]$

span $\{[n], [n]\}$ dimension is 2

span $\{[n], [n]\}$ are in $[R]$

Example.

Find the basis and dimension of the solution space of the following system of homogeneous linear equations:

$$\begin{array}{c|c} x_1+3x_2+2x_3=0\\ 2x_1+8x_2+6x_3=0\\ -4x_1+x_2+3x_3=0 \end{array}$$

$$\begin{array}{c|c} \lambda_3 & \geq 0\\ 2&8&6&0\\ -4&1&3&0 \end{array} \sim \begin{bmatrix} 1&3&2&0\\ 0&1&0&0\\ 0&0&4&0 \end{bmatrix}$$

$$\begin{array}{c|c} \lambda_3 & =0\\ \chi_1 & =0\\ \chi_1 & =0\\ \end{array}$$

$$\begin{array}{c|c} \lambda_1 & =0\\ \end{array} \qquad \begin{array}{c|c} \left\{ \begin{bmatrix} 0\\0\\0\\0 \end{bmatrix} \right\}\\ \text{dimension is } 0 \end{array}$$

$$\hat{\chi} = t \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \hat{\chi} = t \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$